International Journal of Management and Commerce Innovations ISSN 2348-7585 (Online)
Vol. 3, Issue 2, pp: (15-27), Month: October 2015 - March 2016, Available at: www.researchpublish.com

Forecasting Inflation in Kenya Using Arima - Garch Models

Charline UWILINGIYIMANA[1], Joseph MUNGA'TU[2], Jean de Dieu HARERIMANA[3]

[1,2] Jomo Kenyatta University of Agriculture and Technology, [3] Kigali institute of Management

Abstract: **The aim of this study was to empirically develop ARIMA-GARCH models for Kenya inflation and to forecast the rates of inflation using the historical monthly data from 2000 to 2014.**

The empirical research employs time series analysis, ordinary least square and auto-regressive conditional heteroscedastic to find the estimators.

The forecasting inflation analysis have been conducted using two models, the ARIMA (1, 1, 12) model was able to produce forecasts based on the stationarity test and history patterns in the data compared to GARCH (1,2) model. The empirical results of 180 monthly data series indicate that the combination between ARIMA(1,1,12)-GARCH(1,2) model provide the optimum results and effectively improved estimating and forecasting accuracy compared to the other previous methods of forecasting.

Keywords: **Inflation, ARIMA, GARCH, model, time series, modeling and forecasting.**

1. INTRODUCTION

Kenya has experienced strong economic growth for over nearly a decade. However, inflation, which was thought to be under control, has become a major challenge. High, and volatile, inflation is a threat to good economic performance and has negative effects on many of the poor. Economic growth took off in 2004 in Kenya, but alongside higher growth there has been rapid inflation and large inflation volatility. The rise of inflation in Kenya is not an isolated event; other African countries are facing the same problem (AFDB, 2011). Yet, there is no consensus on the causes of the rise in inflation.

In Kenya, the inflation pressures, however, seem to be ticking up in the first half of 2014, pointing to the need for policy makers to keep close watch to ensure that price stability is maintained. A combination of factors notably, rising global crude oil prices, erratic weather patterns that adversely impacted agriculture, and weakening domestic currency as a result of impairment in current account deficits, underlie the evolving inflation developments.

Forecasting inflation is especially challenging in emerging markets given the changes in the underlying environment due to structural and institutional adjustments, and the high weight of volatile food and energy in the consumer price index, and volatility in food and energy prices and the exchange rate.

Modelling and forecasting inflation are generally desirable in an economy for sustaining price rises regardless of source, lead to a fall in real wages and to a distribution of income in favour of profits and low paid workers not protected by trade unions tend to suffer most. To control and maintain inflation often has an adverse effect on balance of payment of a country's current and capital account and thereby aggravates the foreign exchange constraint on development

The purpose of this study is to forecast inflation using univariate time series models, ARIMA and GARCH models. Forecasts of inflation are important because they affect many economic decisions. Without knowing future inflation rates, it would be difficult for lenders to price loans, which would limit credit and investments in turn have a negative impact on the economy.

2. LITERATURE REVIEW

In general, there are two types of approaches for modeling the inflation: macro-economic based models and option pricing based models. In this thesis the latter ones will be applied: geometric Brownian motion for consumer price index and extended Vasicek model for inflation rate.

Cunningham, Hong, & Vilasuso (1997) discovered that the positive relationship between inflation uncertainty and unemployment is dependent on three significant factors. First, the existence of a positive relationship between inflation and unemployment only begins to manifest in mid-1970s. Second, the inflation uncertainty-unemployment relationship is not applicable in every single digit SIC firms. Thirdly, the relationship between inflation uncertainty and unemployment exists only on low-frequency components.

Bruno & Easter (1995) further argued that the negative long-run relationship between inflation and growth found in the above literature is only present with high frequency data and with extreme inflation observations. Batini & Yates (2003) investigated the properties of monetary regimes that combine price-level and inflation targeting. They considered both, at an optimal control and a simple rule characterization of these regimes.

Fama & Gibbons (1997)compared the accuracy of survey respondents' inflation forecast relative to univariate time series modeling on the real interest rate. They observed that the interest rate model yields inflation forecast with a lower error variance than a univariate model, and that the interest rate model's forecast dominate those calculated from the Livingston survey.

Again Ling & Li (1997) considered fractionally integrated moving Average (FIMA) models with conditional heteroscedasticity, which combined with popular generalized auto regressive conditional heteroscedasticity (GARCH) and (ARIMA) models. This is supported by Drost and Klaassen (1997) who argued that financial data set exhibit conditional heteroscedasticity and as a result GARCH – type model are often used to model this phenomenon.

Meyer et al; (1998) considered the autoregressive integrated moving average (ARIMA) for forecasting Irish inflation and justified that ARIMA models are surprisingly robust with respect to alternative (multivariate) model. Gudmundsson (1998) also used the Variable regression coefficient time lags as the source of randomness to find the relationships between economic time series. This was modelled here by means of variable regression coefficients. The model entails heteroscedastic residuals with a negative serial correlation and can be estimated by the Kalman filter.

Most studies on inflation in Kenya do not explicitly deal with the role of food prices for example; (Kiptui, 2009) focuses on the exchange rate and oil prices using a generalized Phillips curve. The study by (AFDB, 2011) also reports that monetary expansion is a key driver of inflation in Kenya, but it only accounts for 30% of the variation in the long run. In fact, the exchange rate seems to explain a large part of the variation according to its coefficient, but no details are provided.

The most recent study on Kenya is (IMF, 2012b) which reports results from work in progress on a small monetary model with Kenya-specific features. The parameters are calibrated, not estimated, which allows for a more complex model specification. The imported food price shocks and poor harvests explain some of the inflation dynamics, both in 2008 and 2011.

The prevailing view in mainstream economics is that inflation is caused by the interaction of the supply of money with output and interest rates (Odedokun, 1993; Stiglitz & Greenwald, 2003). A variety of models and empirical methods have been used in attempts to analyze inflation dynamics.

Assis, Amran and Remali (2006) compared the forecasting performance of different time series methods for forecasting cocoa bean prices at Bagan Datoh cocoa bean. Four different types of univariate time series models were compared namely the exponential smoothing, autoregressive integrated moving average (ARIMA), generalized autoregressive conditional heteroscedasticity (GARCH) and the mixed ARIMA/ GARCH models.

Kontonikas (2004) analyzed the relationship between inflation and inflation uncertainty in the United Kingdom from 1973 to 2003 with monthly and quarterly data. Different types of GARCH Mean (M)-Level (L) models that allow for simultaneous feedback between the conditional mean and variance of inflation were used to test the relationship.

They found that there was a positive relationship between past inflation and uncertainty about future inflation, in line with Friedman and Ball (2006) to which in their study of testing for rate of dependence and asymmetric in inflation uncertainty they concluded that there was a link between inflation rate and inflation uncertainty.

Jehovanes (2007) studied a time lag between a change in money supply and the inflation rate response. A modified generalized autoregressive conditional heteroscedasticity (GARCH) model was employed to monthly inflation data for the period 1994 to 2006: In the study he used the maximum likelihood estimation technique to estimate parameters of the model and to determine significance of the lagged value.

Junttila (2001) applied the Box and Jenkins (1976) approach to model and forecast Finnish inflation. Also, Pufnik & Kunovac (2006) applied the similar approach to forecast short term inflation in Croatia. In many researches in the area of forecasting, the Box & Jenkins (1976) models tends to perform better in terms of forecasting compared to other well-known time series models.

Appiah & Adetunde, (2011) used the Box and Jenkins (1976) approach to model and forecast the exchange rate between the Ghana cedi and the US dollar. In their study, they found that ARIMA (1, 1, 1) model was appropriate for forecasting, the exchange rate.

3. METHODOLOGY

The empirical part of the research sought to validate the model through descriptive and econometrical research designs. The empirical research was sampling and collection of relevant data and the study employs time series analysis with different econometric concept to achieve the empirical results for the model, where OLS and ARCH methods were used to find the estimators.

3.1 ARIMA Model:

The ARIMA model is a combination of two univariate time series model which are Autoregressive (AR) model and Moving Average (MA) model. This model is to utilize past information of a time series to forecast future values for the series.

The ARIMA model with its order is presented as ARIMA (*p*,*d*,*q*) model where *p*, *d*, and *q* are integers greater than or equal to zero and refer to the order of the autoregressive, integrated, and moving average parts of the model respectively. The first parameter *p* refers to the number of autoregressive lags (not counting the unit roots), the second parameter *d* refers to the order of integration that makes the data stationary, and the third parameter *q* gives the number of moving average lags. (Hurvich & Tsai, 1989; Kirchgässner & Wolters, 2007; Kleiber & Zeileis, 2008; Pankratz, 1983; Pfaff, 2008)

A process,$\{y_t\}$ is said to be ARIMA (*p*,*d*,*q*) if $\Delta^d y_t = (1-L)^d y_t$ is ARMA(*p*,*q*).

In general, we will write the model as:

$$\emptyset(L)(1-L)^d y_t = \theta\varepsilon_t; \{\varepsilon_t\} \sim WN(0,\sigma^2)$$

Where ε_t follows a white noise (WN).

$\Delta$is the difference operator

Here, we define the Lag operator by $L^k y_t = y_{t-k}$ and the autoregressive operator and moving average operator are defined as follows:

$$\emptyset(L) = 1 - \emptyset_1(L) - \emptyset_2(L^2) - \cdots - \emptyset_P(L^P)$$

$$\theta(L) = 1 + \theta(L) + \theta_2(L^2) + \cdots + \theta_P(L^P)$$

The functions Ø and θ are the standard autoregressive (AR) and moving average (MA) polynomials of order p and q in variable L, $\emptyset(L) \neq 0$ for$|\emptyset| < 1$, the process $\{y_t\}$ is stationary if and only if $d = 0$, in which case it reduces to an ARMA(p,q) process.

Table 1: Distinguishing characteristics of ACF and PACF for stationary processes

PROCESS	ACF	PACF
AR	Tails off towards zero (exponential decay or damped sine wave)	Cuts off to zero (after lag p)
MA	Cuts off to zero (after lag q)	Tails off towards zero (exponential decay or damped sine wave)

3.1.1 Model Selection Criteria:

The final model will be selected using a penalty function statistics such as Akaike Information Criterion (AIC or AICc) or Bayesian Information Criterion (BIC). (Sakamoto, Ishinguro, & Kitagawa, 1986);(Akaike, 1974) and (Schwarz, 1978). The AIC, AICc and BIC are a measure of the goodness of fit of an estimated statistical model. Given a data set, several competing models may be ranked according to their AIC, AICc or BIC with the one having the lowest information criterion value being the best. These information criterion judges a model by how close its fitted values tend to be to the true values, in terms of a certain expected value.

The criterion attempts to find the model that best explains the data with a minimum of free parameters but also includes a penalty that is an increasing function of the number of estimated parameters.

Also some forecast accuracy test between the competing models can also help in making a decision on which model is the best. Minimum of free parameters but also includes a penalty that is an increasing function of the number of estimated parameters.

This penalty discourages over fitting. In the general case, the AIC, AICc and BIC take the form as shown below: $AIC = 2k - 2\log(L) \quad or \quad 2k - n\log\left(\frac{RSS}{n}\right)$

$$AIC_C = AIC + \frac{2k(k+1)}{n-k-1}$$

$$BIC = -2\log(L) + k\log(n) \quad or \quad \log(\sigma_e^2) + \frac{k}{n}\log(n)$$

Where

k: is the number of parameters in the statistical model

L: is the maximized value of the likelihood function for the estimated model

RSS: is the Residual Sum Squares for the estimated model

n : is the number of observations

σ_e^2:is the error variance

The AICc is a modification of the AIC by Hurvich and Tsai (1989) and it is AIC with a second order correction for small sample sizes. Burnham & Anderson (1998) insist that since AICc converges to AIC as *n* gets large, AICc should be employed regardless of the sample size.

3.1.2 Forecasting using ARIMA model:

The last step in Box-Jenkins model building approach is forecasting. After a model has passed the entire diagnostic test, it becomes adequate for forecasting. Forecasting is the process of making statements about events whose actual outcomes have not yet been observed. In ARIMA models as described by several researchers have proved to perform well in terms of forecasting as compare to other complex models.

One of the most popular univariate forecasting model proposed by Box and Jenkins (1970). For a stationary time series Y_h an ARMA (p, q) model is expressed as

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots.. + \phi_p y_{t-p} + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \ldots. + \theta_p\varepsilon_{t-p}$$

Where ε_t, is a white noise disturbance term normally and independently distributed with mean 0 and variance σ^2. This model can be expressed as weighted sum of disturbances ε_t, as

$$y_t = \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + ... + \theta_k \psi_{t-k}$$

Where ψ weights are functions of the modal parameters ϕ 's and θ 's. An h-step ahead forecast error variance FEV (h) for y is given by

$$FEV(h) = (1 + \psi_1^2 + \psi_2^2 + ... + \psi_{h-1}^2)\sigma^2$$

A 95% forecast confidence interval for h-step ahead forecast is given by

$$\hat{y}_{t+h} \pm 1.96\sqrt{FEV(h)}$$

To choose a final model for forecasting the accuracy of the model must be higher than that of all the competing models. The accuracy of the models can be compared using some statistic such as mean error (ME), root mean square error. (RMSE), mean absolute error (MAE), mean percentage error (MPE), mean absolute percentage error (MAPE) etc. A model with a minimum of these statistics is considered to be the best for forecasting.

3.2 The GARCH Model:

The Generalized ARCH (GARCH), as developed by Bollerslev, (1986), is an extension of the ARCH model similar to the extension of an AR to ARMA process. There are a variety of extensions of the ARCH family of models that include the Exponential GARCH (EGARCH), the Integrated GARCH (IGARCH) (Amos, 2009). These will not be discussed in this research study. For interested reader, thesis by Talke (2003) is good source of such information. The GARCH (p, q) model employs the same equation as ARCH (1,1) for the log-returns r_t but the equation for the volatility, includes q new terms, that is

$$r_t = \sigma_t \varepsilon_t \, , \, \varepsilon_t \sim N(0,1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \cdots + \alpha_q r_{t-q}^2 + \beta_1 r_{t-1}^2 + \cdots + \beta_p r_{t-p}^2$$

Where now $t > \max(p, q)$ and the remaining components are as in the ARCH model. The parameters of the model are $\alpha_0, \alpha_1, \dots, \alpha_q, \beta_1 \dots \beta_p$ for some positive integers p, q .

We see that if $p = 0$ the above model is reduced to the ARCH (q). Thus the GARCH model generalizes the ARCH by introducing values of σ_{t-1}^2, σ_{t-2}^2 …. in the equation: Let $\{r_t\}$ be the mean corrected return, ε_t be a Gaussian white noise with mean zero and unit variance. Let also $H_t$be the information set or history at time t given by $H_t = \{r_1, r_2, \dots, r_{t-1}\}$ as in the ARCH model. Then the process $\{r_t\}$ is GARCH (1,1) if

$$r_t = \sigma_t \varepsilon_t \, , \, \varepsilon_t \sim N(0,1)$$

and $$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 r_{t-1}^2$$

3.2.1 Forecasting with GARCH (p, q) model:

Forecasting using the GARCH model is the same as using the ARMA model. Thus the conditional variance of $\{r_t\}$ is obtained simply by taking the conditional expectation of the squared mean corrected returns. Assuming a forecasting origin of T; then the *l*-step ahead volatility forecast is given by:

$$r_t^2(1) = E[r_{t+1}^2 | r_t]$$

$$= \alpha_0 + \sum_{i=1}^{m} (\alpha_i + \beta_j) E(r_{t+l-i}^2 | r_t) - \beta_j \sum_{i=1}^{p} E(v_{t+l-i} | r_t)$$

Where $r_t^2, \dots, r_{t+l-m}^2 and \sigma_t^2, \dots., \sigma_{t+l-p}^2$are assumed known at time t and the true parameter valuesα_i andβ_j for i = 1… m are replaced by their estimates. Furthermore, the *l*-step ahead forecast of the conditional variance in a GARCH (p, q) model is given by:

$$r_t^2(l) = E[r_{t+l}^2 | r_t]$$

$$= \alpha_0 + \sum_{i=1}^{m} (\alpha_i + \beta_j) E(r_{t+l-i}^2 | r_t) - \beta_j \sum_{i=1}^{p} E(v_{t+l-i} | r_t)$$

Where, $E[r_{t+l}^2 | r_t]$ for $i < l$ can be given recursively as for $i \geq l$, $E(v_{t+l-i} | r_t) = 0$,for $i < l$, $E(v_{t+l-i} | r_t) = v_{t+l-i}$ for $i \geq l$ We now consider the techniques that are used for selecting the best fitting model in line of two or more competing models based on the likelihood ratios.

The accuracy of the models can be compared using some statistic such as mean error (ME), root mean square error. (RMSE), mean absolute error (MAE), mean percentage error (MPE), mean absolute percentage error (MAPE) etc. A model with a minimum of these statistics is considered to be the best for forecasting.

4. RESULTS AND DISCUSSION

This parts presents the analysis and discussion of the results obtained from the study. In this study, a total of 180 monthly inflation data series (month on month-%) is used from January 2000 to December 2014 of month frequencies. The analysis was carried out using both MINITAB 17 and E-views 8.0 statistical software.

4.1 Pre-estimation analysis on inflation rate:

It is recommended that a lengthy time series data is required for univariate time series forecasting. Meyler et al, (1988), recommended that at least 50 observations should be used for such a univariate time series forecasting. The analysis is made on the same data series used by Asadi et al. (2012), Hadavandi et al. (2010), Khashei et al. (2009) and Khashei et al. (2008) .

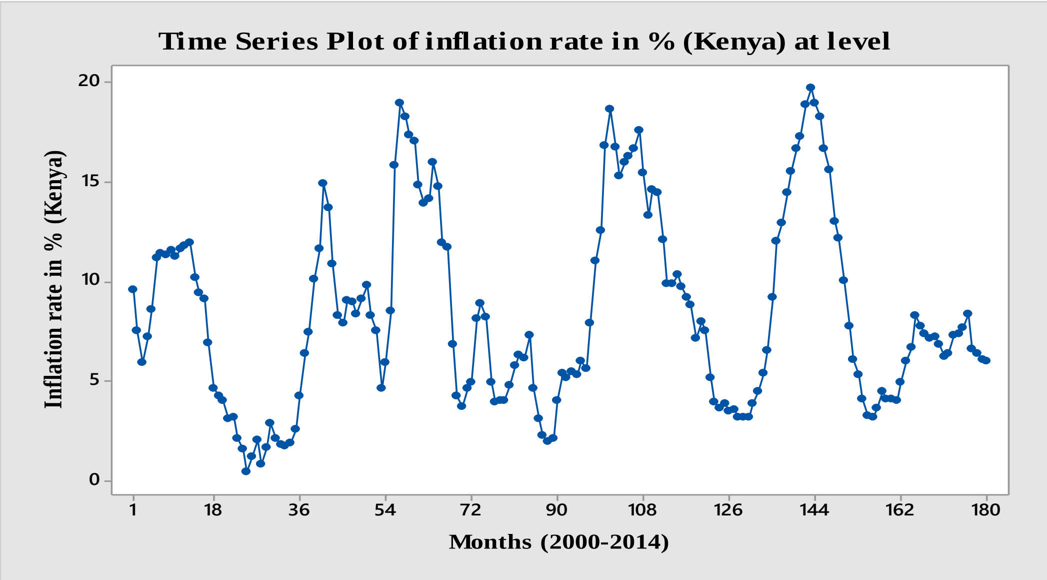

Figure 1: General trend of Kenya Monthly Inflation: period: 2000-2014

It is revealed from figure 1 above that inflation rate for the period of 2000 to 2014 is non-stationary due to an unstable mean which increase and decrease at certain points. The mean and variance ought to be adjusted to form stationary series, so that the values vary more or less uniformly about a fixed level over time. The mean is not constant throughout the series as it assumes a downward trend by decreasing from the highest peak to the lowest peak. There are sudden swings around 2001, 2002, 2003, 2008 and 2011 after which the mean stabilizes in the remaining years whilst the variance reduces from the highest swing it attained, hence the mean and variance are non-stationary.

Table 2: Descriptive Statistics of Inflation (Anderson-Darling Normality Test)

Non-Normal at 0.01		**-2.77292**
A-Squared	3.171	-2.39398
P-value	0.000	-2.20041
95% Critical Value	0.787	-2.06536
99% Critical Value	1.092	-1.95996
Mean	8.392	-1.87269
Standard Deviation	4.784	-1.73166
Variance	22.888	-1.67241
Skewedness	0.597	-1.6185
Kurtosis	-0.606	-1.56892

A normality test performed on the mean and variance using the Anderson-Darling Normality Test at 95% confidence interval see table 2 and figure 2 below, revealed that the mean is rightly skewed with a mean value of 8.392 and variance of 22.888. The coefficient of skewness and kurtosis are 0.597 and -0.606 respectively. It is evident at 5% significant level that there are large swings in the data indicating non stationarity for the period under study.

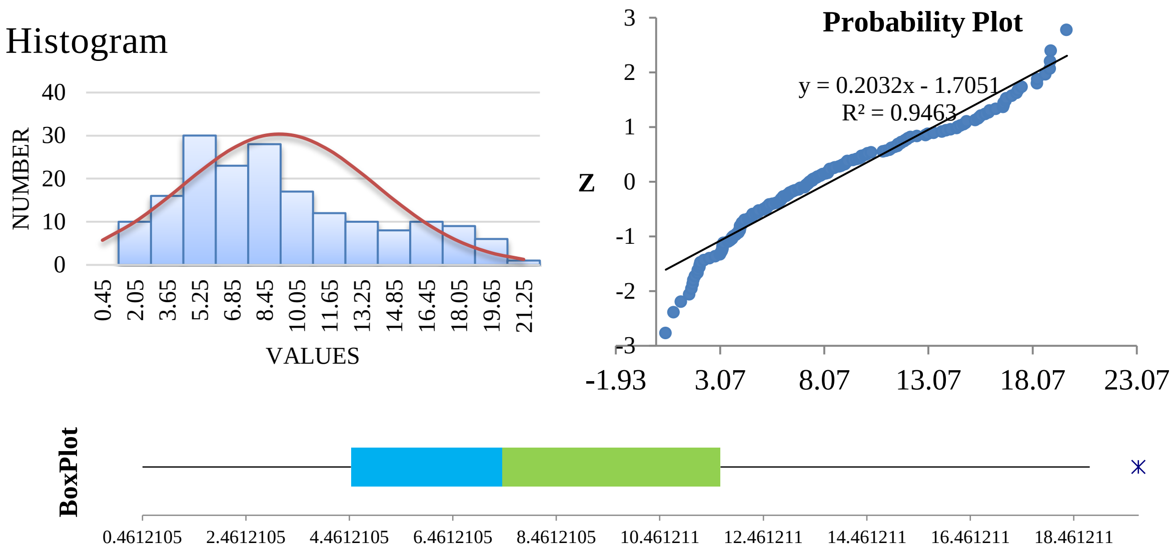

Figure 2: Anderson –Darling Normality Plot for inflation from 2000-2014.

Due to the non-stationarity of the data above which we observe from the time series plot and the Anderson Darling Normality test we also apply the unit root test and precisely the Augmented Dickey Fuller Test

4.2 Univariate time series analysis:

4.2.1 Stationarity tests using Augmented Dickey Fuller (ADF):

Table 3: Stationarity tests using Augmented Dickey Fuller (ADF)

Variable	Equation	At Level			At First Difference			Order of Integration
		t-stat	t-ADF*	P-val.	t-stat	t-ADF*	P-val.	
INFR	Intercept	-2.703	-3.469 -2.878 -2.576	0.075	-6.628	-3.469 -2.878 -2.576	0.000	I(1)
	Intercept & trend	-2.778	-4.013 -3.436 -3.142	0.207	-6.607	-4.013 -3.436 -3.142	0.000	I(1)

t-ADF*: Augmented Dickey-Fuller test critical values at 1%, 5% and 10%

To confirm the presence of stationarity, the Augmented Dickey-Fuller (ADF) test was performed. The test fails to reject the null hypothesis of unit root at 5% level of significance and thus it can be concluded that the rate of inflation is not stationary. For this purpose, a first order lagged difference from the original series is obtained. Augmented Dickey-Fuller (ADF) test is conducted on this series to check for stationarity. The ADF test shows that the series is stationary. The t-statistic of -6.607 and -6.628 is smaller than 1% of test critical value. The p-value for ADF test is zero indicating that we have sufficient evidence to reject the null hypothesis of the series being non-stationary.

4.3 ARIMA model:

4.3.1 Model selection on ARIMA:

Table 4: Different ARIMA (p, d, q) model fitted

ARIMA	AIC	BIC	Log likelihood
(1,0,1)	3.442	3.492	-305.1078
(2,0,2)	4.55	4.61	-402.7092
(3,0,1)	4.384	4.438	-385.0504
(1,0,13)	3.514	3.568	-311.587
(1,1,1)	3.447	3.5	-303.8127
(1,1,12)*	2.88*	2.933*	-253.3484*

*Best based on the model selection criterion

This suggests the use of an Autoregressive Integrated moving-average of order (p, d, q), (ARIMA (p, d, q)). On this account ARIMA (1, 1, 12) model is suggested for tentative model selection. It is found that the AIC value for the model ARIMA (1, 1, 12) is minimum, reflecting the intention to the seasonality test for future analysis. This model includes one AR coefficients and twelve MA coefficient and takes the form: $Infr_t = C + \theta Infr_{t-1} + \phi \varepsilon_{t-12}$

4.3.2 Estimating equation using ARIMA (1, 1, 12):

Table 5: Significance test of the parameters of ARIMA (1, 1, 12)

Dependent Variable: DINFR
Method: Least Squares
Sample (adjusted): 2000M03 2014M12
Included observations: 178 after adjustments
Convergence achieved after 10 iterations
MA Backcast: 1999M03 2000M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.005524	0.030796	-0.179371	0.8579
AR(1)	0.427337	0.068262	6.260212	0.0000
MA(12)	-0.937135	0.018883	-49.62798	0.0000

R-squared	0.561458	Mean dependent var	-0.008427
Adjusted R-squared	0.556446	S.D. dependent var	1.520948
S.E. of regression	1.012949	Akaike info criterion	2.880319
Sum squared resid	179.5615	Schwarz criterion	2.933945
Log likelihood	-253.3484	Hannan-Quinn criter.	2.902066
F-statistic	112.0248	Durbin-Watson stat	1.907864
Prob(F-statistic)	0.000000		

From table 5, the values C, θ and ϕ corresponding to the coefficient of (AR, MA) less than 0.05, which leads to the conclusion that this parameter is significant. Through tests of unit root, autocorrelation and partial correlation coefficients of the corresponding sequence, There is making sure the dynamic model of inflation level (Infr) is ARIMA(1,1,12). Eventually we get the following ARIMA model.

$$Infr_t = -0.0055 + 0.4273 Infr_{t-1} - 0.93713 \varepsilon_{t-12}$$

The reciprocals of all AR root and MA root are less than 1, and this indicates that the ARIMA model is steady. Model residual sequence is also stationary. All these show that the ARIMA method is effective, so it can be used to forecast the future level of inflation in Kenya.

4.3.3 Residual diagnostic for ARIMA model:

Table 6: Breusch-Godfrey Serial Correlation LM Test

F-statistic	1.584644	Prob. F(2,173)	0.2080
Obs*R-squared	3.016046	Prob. Chi-Square(2)	0.2213

The result of the Breusch-Godfrey test for serial autocorrelation shows that Prob (F-stat) is 0.2080 while Prob (Obs*R2) is 0.2213. All these probabilities are greater than 0.05 implying that we cannot reject the null hypothesis instead we reject the alternative hypothesis, and then accept the null hypothesis which states that there is no serial autocorrelation in the model. Based on this, we conclude that there is no autocorrelation of error terms in the model.

Table 7: Heteroskedasticity Test: ARCH

F-statistic	0.009633	Prob. F(1,175)	0.9219
Obs*R-squared	0.009743	Prob. Chi-Square(1)	0.9214

The ARCH Test also negates the presence of autoregressive conditional heteroskedasticity. The result of test for the heteroskedasticity shows that Prob (F-stat) is 0.9218 while Prob (Obs*R2) is 0.9214.

4.3.4 Forecasting using ARIMA (1, 1, 12):

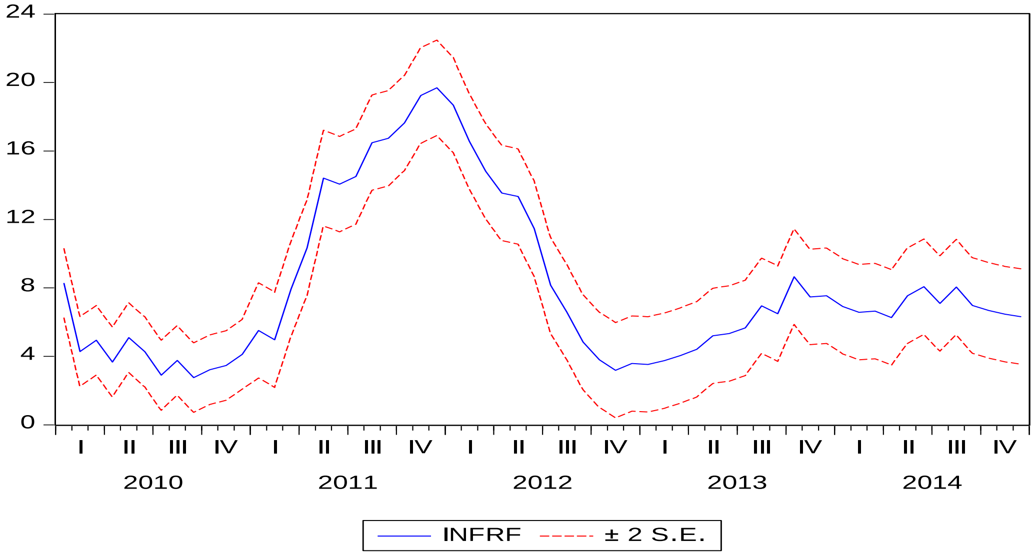

Figure 1: Forecasting the inflation using ARIMA (1, 1, 12)

The duration of forecasts is from January 2010 to December 2014. In the figure 6 the solid line represents the forecast value of inflation rate (2010-2014). Meanwhile, the dotted lines which are above or below the forecasted inflation rate show the forecast with ±2 of standard errors.

4.3.3 GARCH Model:

4.3.3.1 Model selection and analysis:

The idea is to have a parsimonious model that captures as much variation in the data as possible. Usually the simple GARCH model captures most of the variability in most stabilized series. Small lags for p and q are common in applications.

Some models are typically adequate in different study such as GARCH (1, 1); GARCH (2, 1) or GARCH (1, 2) models for modelling volatilities even over long sample periods (Bollerslev, Chou and Kroner, 1992). However in the table 8 below we have included GARCH (0, 1); GARCH (0; 2) and GARCH (2; 2) in order to check if they are appropriate for modelling time varying variances of our data. In the Table 8 below the smaller the AIC and BIC the better. Larger AICs; BICs and standard error makes the model unfavorable.

Table 8: Comparison of suggested GARCH models

Model	AIC	BIC	SE	Log Likelihood
GARCH(0,1)	5.956	6.009	4.784*	-533.073
GARCH(1,1)	5.324*	5.395*	5.285	-475.18
GARCH(0,2)	5.749	5.820	4.945	-513.43
GARCH(1,2)	5.329**	5.417**	5.305	-474.61

The Table 8 shows the competing models to the data with their respective AIC; BIC and SE: From our derived models, using the method of maximum likelihood, the estimated parameters of the models with their corresponding standard error and other statistical tests.

The standard errors are used to assess the accuracy of the estimates, the smaller the better. The model fit statistics used to assess how well the model fit the data are the AIC and BIC: The corresponding values are: AIC = 5.329 and BIC = 5.417 with the log likelihood value of -474.61.

The standard errors are quiet small suggesting precise estimates. Based on 95% confidence level, the coefficients of the GARCH (1; 2) model are significantly different from zero and the estimated values satisfy the stability condition.

4.3.3.2 Estimating Inflation rate using GARCH model:

From Table 9 for the conditional mean equation, the parameter found is μ = 6.105719. The standard normal distribution Z-test has rejected the parameter coefficients equal to zero, while the conditional variance equation gives α_0 = 0.728422, α_1 = 1.183641, β_1 = -0.303085 and β_2 = 0.268150. A high value of β_1 means that volatility is persistent and it takes a long time to change. A high value of α1 means that volatility is spiky and quick to react to market movements (Dowd, 2002).

Somehow, R^2 gives a negative value in the estimation equation. In reality, the measure of R^2 in GARCH model is not important because it is only used to test the ARCH effect of residuals. The GARCH (1, 2) model can be written into conditional mean and conditional variance Equations as:

Table 9: Estimating equation using GARCH (1, 2) -Normal distribution

Dependent Variable: INFR
Method: ML - ARCH (Marquardt) - Normal distribution
Sample: 2000M01 2014M12
Included observations: 180
Convergence achieved after 88 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1) + C(5)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	6.105719	0.114874	53.15152	0.0000
	Variance Equation			
C	0.728422	0.329824	2.208518	0.0272
RESID(-1)^2	1.183641	0.181646	6.516202	0.0000
GARCH(-1)	-0.303085	0.057376	-5.282456	0.0000
GARCH(-2)	0.268150	0.041233	6.503240	0.0000

R-squared	-0.229721	Mean dependent var	8.392357
Adjusted R-squared	-0.229721	S.D. dependent var	4.784168
S.E. of regression	5.305298	Akaike info criterion	5.329008
Sum squared resid	5038.167	Schwarz criterion	5.417701
Log likelihood	-474.6107	Hannan-Quinn criter.	5.364969
Durbin-Watson stat	0.082156		

$$Infr_t = 6.1057 + \varepsilon_t$$

$$\sigma_t^2 = 0.7284 + 1.1183\varepsilon_{t-1}^2 - 0.303\sigma_{t-1}^2 + 0.268\sigma_{t-2}^2$$

4.3.3.3 Post-estimation of the GARCH Model:

From model selection criteria, the model which has minimum Akaike Information Criteria (AIC), and Schwartz Bayesian Criteria (SBC) value, is the best model.

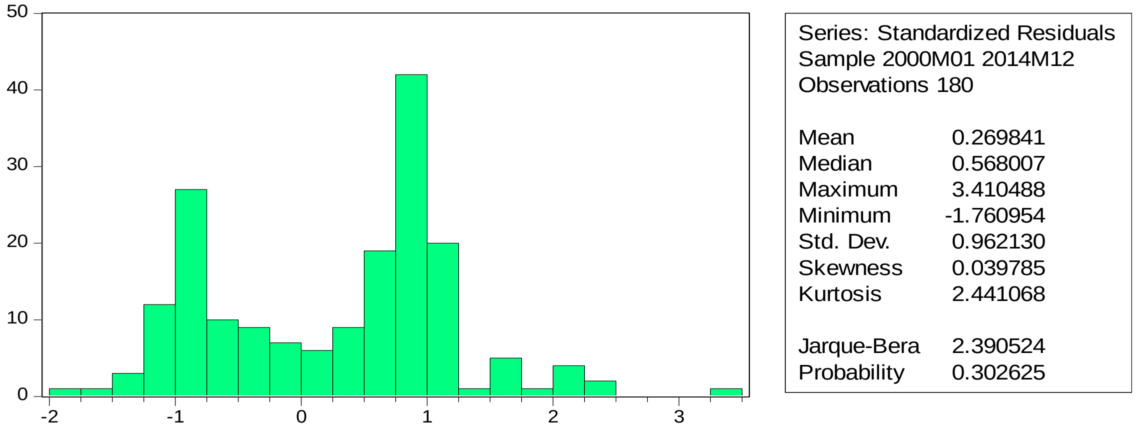

Figure 2: Normality test for GARCH model

The result of the Normality test shows that Jarque-Bera value is 2.390 with a probability of 0.3026, this probability value, however is more than 0.05 meaning that we cannot reject the null hypothesis, instead we reject the alternative hypothesis and fail to reject the null hypothesis which states that the residual is normally distribute. Based on this however we conclude that the residual is normally distributed.

Table 10: Heteroskedasticity test: ARCH

F-statistic	12.28671	Prob. F(1,177)	0.0006
Obs*R-squared	11.61899	Prob. Chi-Square(1)	0.0007

In diagnostic checking stage, a test for presenting of conditional heteroscedasticity in the data with ARCH-LM test on the residuals. There is computed one lag difference from the residuals squared in the ARCH-LM test. The test is tabulated (Table 10). The ARCH-LM for one lag difference of residuals squared is 11.6189 under. However, the null hypothesis is rejected the homoscedasticity since the p-value has less than 5% of significance level.

On the other hand, F-statistic for the test is 12.286 also rejected the null hypothesis at the same condition. The ARCH-LM test on the residuals of this model indicates that the conditional heteroscedasticity is present in the data.4.3.3.4 Forecasting using GARCH (1, 2):

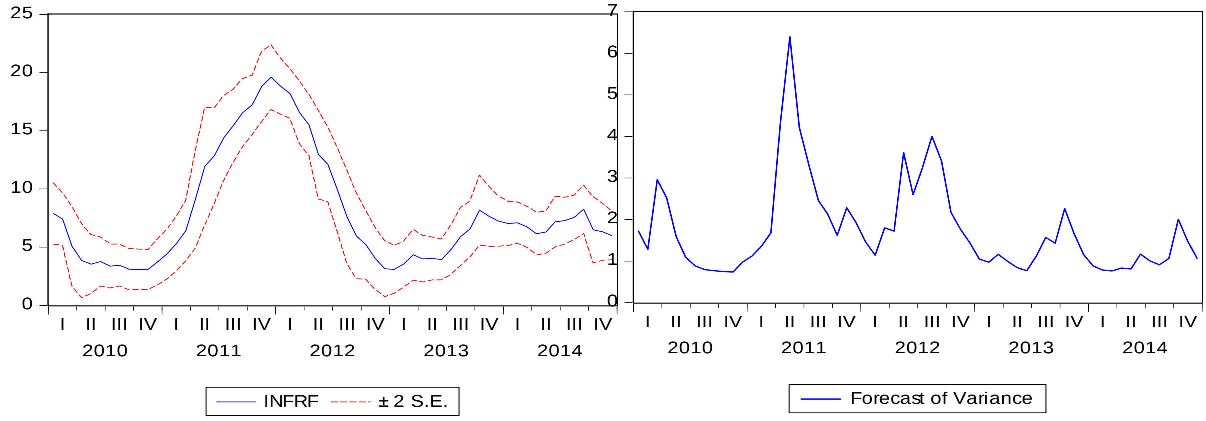

Figure 3: Forecasting the inflation rate using GARCH (1, 2)

Apart from forecasting the conditional variance, the forecast of the conditional mean is done at the same time. Here, the daily forecast inflation are the conditional mean from the original series. Figure 9 shows the forecast value for inflation rate of Kenya using GARCH (1, 2) model. In figure 9 the blue line presents the forecasted inflation whereas the dotted (red) lines are forecast inflation with ±2 standard errors.

The forecast of conditional variance is plotted in right of the figure 9. As shown in figure 9the forecast of conditional variance is not constant. Since conditional heteroscedasticity searches for the non-constant variance that exists in time series data, then its trend is non-linear.

4.3.4 Forecasting comparison using ARIMA and GARCH models:

Empirically taking, we have examined that table 11 reports the various measures of forecasting errors, namely the mean absolute error (MAE); the root mean squared error (RMSE); and Thieles U for two models seemed to be adequately fit the data. The first two forecast error statistics depend on the scale of the dependent variable. These are used as relative measure to compare forecasts for the same series across different models, the smaller the error the better the forecasting ability of that model accordingly. The remaining two statistics are scale invariant. The Theil inequality coefficient always lies between zero and one, where zero indicates a perfect fit.

Table 11: Forecasting comparison using ARIMA - GARCH models

Models	**Inflation rates in Kenya**				
	RMS	Theil	MAE	MAPE	BIAS
ARIMA (1,1,12)	0.7326	0.0391	0.5868	8.7164	0.000084
GARCH (1,2)	1.113	0.059	0.8530	11.5017	0.005

In the forecasting stage, Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Theil Inequality Coefficient (Theil-U) values for ARIMA (1, 1, 12) and GARCH (1,2) models are determined. These are tabulated in Table 11. If the actual values and forecast values are closer to each other, a small forecast performance were obtained. Thus, smaller RMSE, MAE, MAPE and Theil-U values are preferred.

From Table 11, it can be concluded that all forecast performance from GARCH (1, 2) model is greater than that from ARIMA (1, 1, 12) model. Therefore, we can conclude that ARIMA (1, 1, 12) model performs better than GARCH (1, 2). In other words, ARIMA (1, 1, 12) is a better forecast model for inflation rate than GARCH (1, 2) model.

5. CONCLUSION

There are many instances where additional knowledge pertaining to forecast variances derived from a GARCH process could be beneficial. In addition, the normality assumption associated with the conditional distribution does not present a limitation.

The stages in the model building (that is the identification, estimation and checking) strategy has been explored and utilized. Based on minimum AIC and BIC values, the best _t GARCH models tend to be GARCH (1, 1) and GARCH (1, 2) . The GARCH (1, 1) model has smaller AIC and BIC which is an indicative that it explains the data better than GARCH (1, 2) model.

This study examined the performance of combination of the most powerful univariate time series, ARIMA models with the superior volatility models, GARCH in analyzing and forecasting monthly inflation rate data series. The Box-Cox formula is used in the data transformation step to address non stationarity in variance. The empirical results of 180 monthly data series indicate that the combination between ARIMA(1,1,12)-GARCH(1,2) model provide the optimum results and effectively improved estimating and forecasting accuracy compared to the ten previous methods of forecasting in literatures. In conclusion, the complete combination of powerful and flexibility of ARIMA and the strength of GARCH models in handling volatility and risk in the data series as well as to overcome the linear and data limitation in the ARIMA models made the combination of ARIMA-GARCH as a new potential approach in analyzing and forecasting inflation rate.

The inflation model obtained is stochastic in nature and is therefore recommended for use by future researcher's as basis for constructing deterministic models such as first order stochastic differential equation, using current economic trend. The model can further be used for prediction and explanation purposes by connecting it to the macroeconomic theory.

REFERENCES

[1] AFDB. (2011). *Inflation Dynamics in Selected East African Countries: Ethiopia, Kenya, Tanzania and Uganda*. Tunisia: AfDB Brief. African Development Bank.

[2] Akaike, H. (1974). A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control*, *19*(6), 716–723.

[3] Appiah, S. ., & Adetunde, I. (2011). Forecasting exchange rate between the Ghana cedi and the US dollar using time series analysis. *Current Research Journal of Economic Theory*, *3*(2), 76–83.

[4] Batini, N., & Yates, A. (2003). Hybrid inflation and price –level targeting. *Journal of Money, Credit and Banking*, *35*(3), 4–52.

[5] Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, *31*, 307–327.

[6] Box, G. E. ., & Jenkins, G. . (1976). *Time Series Analysis, Forecasting and Control*. California, USA: San Francisco, Holden-Day.

[7] Bruno, M., & Easter, W. (1995). *Inflation crises and Long-Run Growth* (Working Paper No. No. 1517). World Bank.

[8] Cunningham, S. ., Hong, T., & Vilasuso, J. . (1997). Time series analysis of the relationship between inflation uncertainty and employment. *Journal of Macroeconomics, University of Connecticut, Storis*, *19*(4), 731–751.

[9] Fama, E. ., & Gibbons, S. . (1997). Asset Returns and inflation. *Journal of Financial Economic Policy*, *5*(2), 115–146.

[10] Fischer, S., & Modigliani, F. (1978). Towards and Understanding of the Real Effects and Costs of Inflation. *Weltwirtschaftliches Archiv*, *114*, 810–833.

[11] Hurvich, C. ., & Tsai, C (1989). Regression and Time Series Model Selection in Small Sample. *Biometrika*, *76*, 297–307.

[12] IMF. (2012). *Kenya: Third Review under the Three-Year Arrangement Under the Extended Credit*. Washington D.C.: International Monetary Fund.

[13] Jehovanes, A. (2007). *Monetary and Inflation Dynamics: A Lag between Change in Money Supply and the Corresponding Inflation Responding in Tanzania* (Monetary and financial affairs, Department). Dar Es Salaam: Bank of Tanzania.

[14] Junttila, J. (2001). Structural breaks, ARIMA model and Finnish inflation forecasts. *International Journal of Forecasting*, *17*, 203–230.

[15] Kiptui, M. (2009). *Oil Price Pass-Through into Inflation in Kenya*. Kenya School of Monetary Studies Research Centre, Nairobi.

[16] Kirchgässner, G., & Wolters, J. (2007). *Introduction to Modern Time Series Analysis*. Verlag Berlin Heidelberg: Springer.

[17] Kleiber, C., & Zeileis, A. (2008). *Applied Econometrics with R*. New York, USA: Springer Science + Business Media, LLC.

[18] Nicas, Y., & Kessy, N. J. (2015). Appropriate Threshold Level of Inflation for Economic Growth: Evidence from the Three Founding EAC Countries. *Applied Economics and Finance*, *2*(3).

[19] Pankratz, A. (1983). *Forecasting with Univariate Box-Jenkins Models: Concepts and Cases*. USA: John Wiley & Sons, Inc.

[20] Pfaff, B. (2008). *Analysis of Integrated and Cointegrated Time Series* (2nd ed.). USA: Springer Science + Business Media, LLC.

[21] Pufnik, A., & Kunovac, D. (2006). *Short-Term Forecasting of Inflation in Croatia with Seasonal ARIMA Processes* (Working Paper No. No. 16). Croatian National Bank.

[22] Sakamoto, Y., Ishinguro, M., & Kitagawa, G. (1986). *Akaike Information Criterion Statistics*. D. Reidel Publishing Company.

[23] Sargent, T. J., & Wallace, N. (1981). *Some Unpleasant Monetarist Arithmetic* (Quarterly Review) (pp. 1–17). Minneapolis: Federal Reserve Bank.

[24] Schwarz, G. . (1978). Estimating the Dimension of a Model. *Annals of Statistics*, *6*(2), 461–464.

[25] Talke, I. . (2003). *Modelling volatility in time series data* (MSc Thesis). University of Kwazulu Natal, South Africa.